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Short Papers

Analysis of Lossy Inhomogeneous Waveguides Using Shooting Methods

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Abstract—**Shooting methods are used to analyze rectangular waveguides containing inhomogeneous lossy dielectrics. The technique obtains the electromagnetic fields inside the waveguide by solving Maxwell's equations using trial and error procedures to match the boundary conditions at the conducting waveguide surface. Dispersion and attenuation curves are obtained which show how continuous dielectric inhomogeneities and losses affect the transmission characteristics of these waveguides.**

I. INTRODUCTION

The many applications of inhomogeneously loaded waveguides in microwave engineering has resulted in a need for methods of calculating the transmission characteristics of the waves that propagate in such waveguides. A number of solution methods have been developed to analyze such problems, most of which are numerical, since only a few inhomogeneous cases can be solved in closed form [1].

Most of the methods developed are restricted to lossless inhomogeneities. Some of the earlier ones [2]–[4] treat waveguides containing one or two slabs of lossless dielectric. Further developments include Galerkin's method and modification thereof [5], [6], analytical approximations [7], Rayleigh–Ritz optimization [8], [1], finite difference [9], finite element (especially helpful for arbitrary waveguide cross sections) [10]–[12], computer iterations [13], vector variational [14], and shooting methods [15].

Rectangular waveguides containing lossy dielectric slabs have also been analyzed [13], [16]–[18]. Perhaps one of the more elegant papers in this area was written by Gardiol [18]. Using a matrix formulation, he treated general waveguides containing linear, inhomogeneous, lossy, and anisotropic slabs. In principle, his formulation is valid for solving waveguides which have any number of slabs extending across them. However, in practice, the number of computer operations prohibits the computation of waveguide propagation constants (even for the isotropic case) when a large number (e.g., more than 25) of slabs are needed to model the medium contained inside the waveguide. This limitation is not very important in those situations where the medium is accurately modeled by a few step discontinuities but it can be serious when treating certain geometries with continuously varying media.

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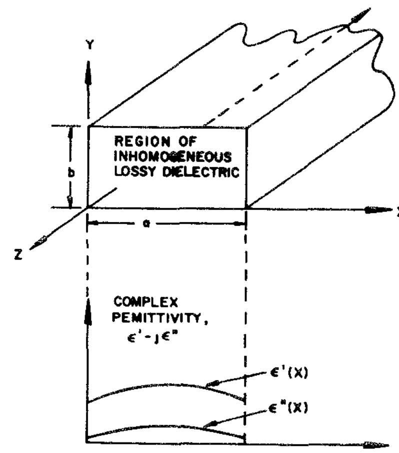

Fig. 1. Rectangular inhomogeneous lossy waveguide with width a and height b. The permittivity $\epsilon(x) = \epsilon'(x) - j\epsilon''(x)$ is a function of the spatial coordinate x.

In this short paper we present an approach which differs from Gardiol's in the technique used to solve Maxwell's equations. We show that the propagation characteristics of a rectangular waveguide loaded with an isotropic, lossy, and inhomogeneous dielectric as shown in Fig. 1 can be found by applying shooting methods directly to the field equations. This technique has the advantage that the permittivity does not have to be approximated by a small number of slabs. The field components are also available for printout and display since they are computed in determining the dispersion curves.

The complex propagation constants for an inhomogeneous rectangular waveguide are obtained by solving the first-order differential equations (Maxwell's equations) using Hammings stable method [19], started by a Runge–Kutta–Gill method. The ability to select the size of the spatial increments used in the iteration procedure further allows this technique to yield good accuracy for higher order modes and strong inhomogeneities of the permittivity [15].

In Section II, Maxwell's equations are formulated appropriately for a rectangular geometry, and a description of the solution technique is given when the inhomogeneity can be expressed as a function of one spatial coordinate. An example is presented in Section III to illustrate the speed and accuracy of the method.

II. THEORY

Maxwell's equations contain all of the necessary information to obtain the wave propagating characteristics of waveguides. We need only solve them inside the waveguide of width a and height b shown in Fig. 1 subject to the appropriate boundary conditions. For one-dimensional inhomogeneities in the x direction, the electric field intensity $\vec{E}$ and the magnetic field intensity $\vec{H}$ of a wave traveling in the $+z$ direction obey Maxwell's equations. Assuming time harmonic fields, they can be written [18]:

$$\nabla_x \begin{bmatrix} \hat{E}_y \\ \hat{E}_z \\ \eta\hat{H}_y \\ \eta\hat{H}_z \end{bmatrix} = \frac{1}{k} \begin{bmatrix} 0 & 0 & \gamma h & (k^2 - h^2) \\ 0 & 0 & -(k^2+\gamma^2) & \gamma h \\ -\gamma & (k^2 - h^2) & 0 & 0 \\ -(k^2+\gamma^2) & -\gamma h & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} \hat{E}_y \\ \hat{E}_z \\ \eta\hat{H}_y \\ \eta\hat{H}_z \end{bmatrix} \quad (1)$$

$$\hat{E}_x = -\frac{\nabla_x \hat{E}_z}{\gamma} - \frac{k\eta\hat{H}_x}{\gamma} \quad (2)$$

and

$$\hat{H}_x = -\frac{\nabla_x \eta\hat{H}_z}{\eta\gamma} - \frac{k\hat{E}_y}{\eta\gamma} \quad (3)$$

where ∇_x denotes differentiation with respect to the spatial coordinate x, $\eta = -j[\mu(x)/\epsilon(x)]^{1/2}$, $k = \omega[\mu(x)\epsilon(x)]^{1/2}$, $\mu(x)$ is the magnetic permeability and $\epsilon(x)$ is the electric permittivity, and $h = n/b$ (n is an integer giving the field variation in the y direction). Furthermore, the terms under the caret sign are functions of x only and are related to the field quantities by the relationship

$$\begin{bmatrix} E_x(x,y,z) \\ E_z(x,y,z) \\ H_y(x,y,z) \end{bmatrix} = \begin{bmatrix} \hat{E}_x \\ \hat{E}_z \\ \hat{H}_y \end{bmatrix} \sin hy \exp(-\gamma z)$$

$$\begin{bmatrix} E_y(x,y,z) \\ H_x(x,y,z) \\ H_z(x,y,)z \end{bmatrix} = \begin{bmatrix} \hat{E}_y \\ \hat{H}_x \\ \hat{H}_y \end{bmatrix} \cos hy \exp(-\gamma z) \quad (4)$$

where $\gamma = \alpha + i\beta$ is the complex propagation constant.

The electromagnetic fields inside a rectangular waveguide can be obtained by simultaneously solving the set (1) by shooting methods [15]. The solution is started by selecting the tangent $\vec{E}$ fields and normal $\vec{H}$ fields to be zero at the $x = 0$ boundary. Furthermore, the normal $\vec{E}$ fields and tangent $\vec{H}$ fields are selected to yield TE or higher order modes as desired. We find that choosing $\hat{H}_y|_{x=0} = 0$ yields TE_{mn} modes,[1] $\hat{E}_x|_{x=0} = 0$ yields LSE modes, and $\hat{H}_x|_{x=0} = 0$ yields LSM modes.

Having chosen appropriate boundary conditions, a search for γ that results in fields that satisfy the boundary conditions at $x = a$ is begun. This search is initiated by selecting starting values for both α and β and computing the electromagnetic fields in the waveguide. If the boundary conditions at $x = a$ are satisfied, the chosen γ is correct because Maxwell's equations are satisfied throughout the region including the boundaries. The uniqueness theorem also guarantees that the fields obtained are the only fields for which the calculated γ is a propagation parameter.

If the boundary conditions are not satisfied, another value of γ is selected. This procedure is repeated in a systematic way (outlined below) until the propagation parameters satisfy all boundary conditions. The search for the complex propagation constant is conducted in the following manner.

1) Initial values of α and β are chosen to initiate the procedure (e.g., for the first frequency, $\alpha_0 = 0$ and $\beta_0 \gtrsim \omega/c$).

2) α is kept constant while two values of β are sought which satisfy the condition $\text{Re}\{\hat{E}_y|_{x=a}\} = E_y'|_{x=a} = 0$ and $I_m\{\hat{E}_y|_{x=a}\} = E_y''|_{x=a} = 0$.

3) α is increased in steps until the two values, β_1 and β_2, obtained from step 2) approximately agree, and bounds established by the condition that

$$|E_y||_{x=a} = (E_y'|_{x=a}^2 + E_y''|_{x=a}^2)^{1/2} < \epsilon$$

are satisfied (where ϵ is a small number to be approximately five orders of magnitude less than the peak value of $|E_y|$ for our example.)

The dispersion and attenuation curves are obtained by repeating the previous steps for various frequencies. Once the value of γ is obtained for a given frequency, the procedure is expedited at the next frequency by using that value to initiate the process.

[1] The integer m is obtained by observing the number of zero crossings which the computed field undergoes.

III. EXAMPLE

To check our procedure we first obtained the fields inside an empty waveguide. Our calculations were done on a CDC 3600/3800 time sharing computer system. By selecting the appropriate boundary conditions, we computed the TM_{11} mode and compared it with the known solution. We obtained four decimal place accuracy for the β when using less than 10 s of computer time. The resulting electromagnetic fields were found to be within 0.1 percent of the analytical solutions. For this example, the final values of the β's were obtained after approximately five tries using the search procedure outlined previously where the accuracy was determined by the choice of ϵ.

A. A Waveguide Example

As an example of a lossy inhomogeneous waveguide we assume a complex permittivity of the form

$$\epsilon(x/a) = \epsilon'(x/a) - j\epsilon''(x/a)$$

where

$$\epsilon'(x/a) = \epsilon_0[1 - 2(x/a)(x/a - 1)]$$

and

$$\epsilon''(x/a) = K\epsilon_0[1 - 2(x/a)(x/a - 1)] \quad (5)$$

where K is a loss factor which we will vary in our analysis. Using the technique of Section II, the TE_{10}, TE_{20}, and LSE_{11} modes were obtained. The EM fields for the preceding modes were observed to be nearly the same as those for the lossless case except for the phase of the components of the fields with respect to each other (i.e., for the lossless case all field components are in phase or 90° out of the phase but this does not hold for a lossy media).

The propagation parameters are plotted as a function of the normalized wavenumber in Figs. 2 and 3. It can be seen that propagation essentially ceases for the two lossy modes shown at frequencies below the cutoff frequencies of the TE_{10} and TE_{20} modes in a lossless inhomogeneous waveguide (where $\epsilon'(x)$ is given by (5) and $\epsilon''(x) = 0$). At higher frequencies, the dispersion of the lossy waveguide is similar to the lossless waveguide (except for a slight increase in the group velocity $\partial\omega/\partial\beta$). The homogeneous loaded waveguide of average permittivity has an even larger group velocity for high frequencies but a smaller group velocity for the lower frequencies. Hence the inhomogeneous loaded waveguide is more dispersive.

To see the effect of increasing losses on α and β, we have also plotted dispersion and attenuation curves for $K = 0.1$. It can also be seen that the phase velocity of the TE_{10} mode surpasses the speed of light in free space for sufficiently low frequencies. However, the loss coefficient $\alpha\lambda$ is observed to be nonzero and strongly dependent on frequency in Fig. 3. Furthermore we compared our results to those for a homogeneous loading having the same average permittivity. We found that the loss coefficient $\alpha\lambda$ of the homogeneous case was slightly less for the high frequencies but that it became larger near the cutoff frequency as illustrated in Fig. 3 for the TE_{01} mode. Optimum transmission frequencies can be observed for both the TE_{10} and TE_{20} modes which are reminiscent of lossy homogeneous transmission lines [1].

This technique has been found useful in obtaining dispersion relationships for waveguides containing continuously varying lossy dielectrics. The solution of such problems are of interest to those engineers working at high microwave frequencies who must deal

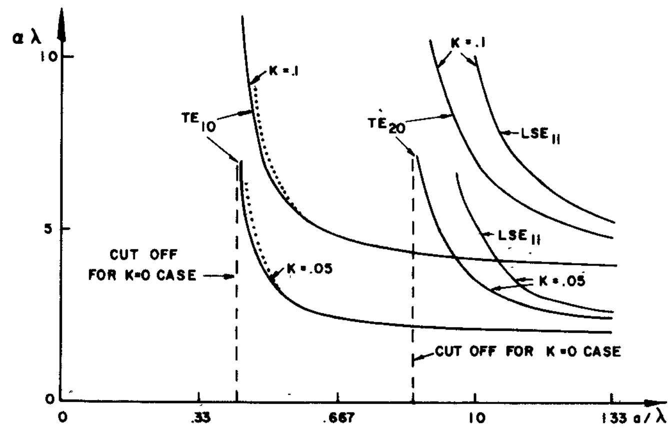

Fig. 2. Dispersion curves for an inhomogeneous lossy dielectric where K is defined by $K = \epsilon''/\epsilon'$. The dotted line represents dispersion curves for the homogeneous loading of average permittivity when $K = 0.1$.

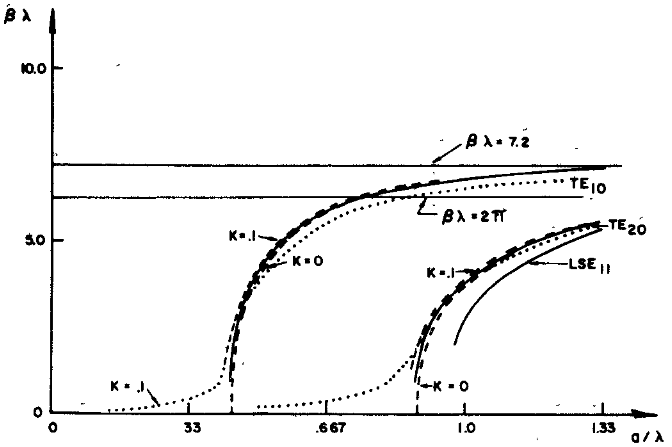

Fig. 3. Loss curves for an inhomogeneous lossy dielectric TE_{10} mode, TE_{20} mode, and LSE_{11} mode. Comparison is made with a homogeneous lossy dielectric (dotted line) for the TE_{10} mode.

with waveguides containing solid-state materials whose dielectric properties vary over distances which cannot be neglected compared to a wavelength. However, for those problems where the dielectric material is accurately modeled by a small number of dielectric slabs, our method is cumbersome owing to the step discontinuities. In those cases, the reader is advised to follow one of the referenced procedures.

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Operation Modes of a Waveguide *Y* Circulator

YOSHIHIKO AKAIWA

***Abstract*—Operation modes of a waveguide *Y* circulator with a circular and a triangular ferrite post are investigated both theoretically and experimentally. Field analysis is carried out taking into consideration the field variation along the ferrite axis. Frequencies are calculated by assuming TM modes nearly agree with measured frequencies. It is shown that the circulator action occurs at frequencies where two HE modes interfere with each other, besides occurring at HE mode resonance frequencies. Effects of Teflon spacers on circulator performances are investigated in detail.**

I. INTRODUCTION

The waveguide *Y* circulator has been widely used in microwave circuits since the first introduction by Chait and Curry [1] in 1959. The design concepts are based on the general theory of the scattering matrix established by Auld [2] and on theories of field analyses by Bosma [3] and Fay and Comstock [4]. These theories are not sufficient, since the scattering matrix theory never shows internal fields of the circulator and the field analysis theory is for stripline circulators.

Determination of operation modes is most important for circulator design, since the circulation occurs at the mode resonance frequencies. Surface wave modes had been considered by Skomal [5] to explain the circulation, however, experiments were not carried out to assert the surfaces wave modes. Little has been known about the waveguide circulator modes for a long time. Recently, Owen [6] first clarified the operation modes of a waveguide *Y* circulator by measuring the eigenvalues. He showed for partial height ferrites that fields vary along the axis of the ferrite and clearly showed that circulator operation is obtained at the resonance frequencies of the ferrite for rotational phase eigen excitations. He identified the ferrite resonance modes as HE_{mnl} modes. The fact that the fields vary along the axis of ferrite has not been taken into consideration in waveguide circulator theories developed before [7], [8].

Although field analysis was carried out taking into consideration the variation along the ferrite post axis for a demagnetized ferrite post, the ferrite resonance phenomena was not recognized to be important for circulator operation and the operation modes were not discussed [9].

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